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Solution of twist-3 evolution equation in double logarithmic approximation in QCD

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Abstract. The solution of the DGLAP evolution equation for the twist-3 gluon operators is obtained in the double logarithmic approximation of QCD perturbation theory. The method used for the solution is similar to the reggeon field theory. The asymptotics of the twist-3 parton correlation function for small Bjorken variables x_{B} is found.

1 Introduction

The growth of the structure function $f(x_{\mathrm{B}}, Q^2)$ in the region of small Bjorken variables x_{B} makes it necessary to take into account the high twist contribution. According to the operator product expansion the local operator of spin J contributes to the number J Mellin moment of the structure function $f(x_{\mathrm{B}}, Q^2)$. The Q^2-dependence of the operators is perturbatively determined by the evolution equation collecting in the leading order (LLA) the terms of the form $(\alpha_{\mathrm{S}} \ln Q^2/\mu^2)^n$. The asymptotic behavior for $x_{\mathrm{B}} \to 0$ is governed by the rightmost singularity of the anomalous dimension in the variable J continued to the complex plane. However, the calculation of the anomalous dimension becomes more complicated for twists $N \geq 3$ because of the large number of local operators.

There is another approach proposed in [1] and based on the relation of the small-x_{B} behavior of the structure function and the BFKL equation summing up the powers $(\alpha_{\mathrm{S}} \ln 1/x_{\mathrm{B}})^n$. It enables one to find the twist-2 anomalous dimension near the singularity position $J \to 1$ for the leading and next to leadings orders in terms of $\ln Q^2$. The situation is also more involved for the higher twists, $N \geq 3$, since one has to solve the equation for N reggeized gluons [2] and then to extract from it the anomalous dimension.

2 Evolution equation for the twist-3 quasipartonic operators in DLA

Here we consider the evolution equation for the twist-3 quasipartonic operators in double logarithmic approximation (DLA), which collects the powers of the product $\alpha_{\mathrm{S}} \ln Q^2/\mu^2 \ln 1/x$. Quasipartonic operators form a closed set of high twist operators allowing for an interpretation in terms of the parton model [5]. They are responsible for the small-x_{B} asymptotics of the structure function [3,4]. The matrix elements of quasipartonic operators depend only on the fraction x_i of the partons momenta along the hadron momentum p ($p^2 \simeq 0$). The pure gluon channel will be studied below as dominating in the small-x_{B} region. We shall take the quasipartonic operators, which, in the axial gauge $n_\mu A_\mu = 0$ with a light-like vector n dual to the hadron momentum p, have the general form

$$
\begin{aligned}
&\mathcal{O}^{m_1,m_2,m_3}_{\mu_1,\mu_2,\mu_3} \\
&= \Gamma^{abc}_{\mu_1\mu_1',\mu_2\mu_2',\mu_3\mu_3'} \left((\mathrm{i}\partial)^{m_1} A^a_{\mu_1'}\right)\left((\mathrm{i}\partial)^{m_2} A^b_{\mu_2'}\right)\left((\mathrm{i}\partial)^{m_3} A^c_{\mu_3'}\right),
\end{aligned}
$$

where m_i are positive integers, Γ is a color and Lorentz tensor, the particular form of which will not be important in what follows; $\partial = n_\mu \partial_\mu$. The matrix element over a hadron state can be expressed as

$$
\begin{aligned}
\langle h|\mathcal{O}^{m_1,m_2,m_3}_{\mu_1,\mu_2,\mu_3}|h\rangle &= \int \mathrm{d}\beta_1 \mathrm{d}\beta_2 \mathrm{d}\beta_3\, N^{abc}_{\lambda_1\lambda_2\lambda_3}(\beta_1,\beta_2,\beta_3) \\
&\quad \times\ \mathcal{O}^{abc}_{\lambda_1\mu_1,\lambda_2\mu_2,\lambda_3\mu_3} \beta_1^{m_1}\beta_2^{m_2}\beta_3^{m_3}, \\
\mathcal{O}^{abc}_{\lambda_1\mu_1,\lambda_2\mu_2,\lambda_3\mu_3} &= \Gamma^{abc}_{\mu_1\mu_1',\mu_2\mu_2',\mu_3\mu_3'} \varepsilon^{\lambda_1}_{\mu_1'}\varepsilon^{\lambda_2}_{\mu_2'}\varepsilon^{\lambda_3}_{\mu_3'},
\end{aligned}
$$

where ε^λ_μ is the gluon polarization vector. The parton correlation function $N^{abc}_{\lambda_1\lambda_2\lambda_3}(x_1,x_2,x_3)$ has the meaning of a hadron wavefunction integrated over the partons' transverse momenta, the greatest transverse momentum being of the order Q^2.

The evolution equation generalizing the twist-2 DGLAP equation is derived in [5]. It has the form of an N-particle one dimensional Schrödinger-type equation with pairwise interaction between the gluons,

$$
\begin{aligned}
&Q^2 \frac{\partial}{\partial Q^2} N^{a_1,\ldots,a_i,a_j,\ldots,a_N}_{\lambda_1,\ldots,\lambda_i,\lambda_j,\ldots,\lambda_N}(Q^2, x_1, \ldots, x_N) \\
&= \sum_{i<j} \int \mathrm{d}\beta_i \mathrm{d}\beta_j\, \delta(x_i + x_j - \beta_i - \beta_j) \qquad (1)
\end{aligned}
$$

$$\times\, \Phi^{a_i a_j, a_i' a_j'}_{\lambda_i \lambda_j, \lambda_i' \lambda_j'}(x_i, x_j; \beta_i, \beta_j)$$
$$\times\, N^{a_1,\dots,a_i',a_j',\dots,a_N}_{\lambda_1,\dots,\lambda_i',\lambda_j',\dots,\lambda_N}(Q^2, x_1, \dots, \beta_i, \dots, \beta_j, \dots, x_N).$$

This equation sums up in the leading $\ln Q^2$ order the ladder-type diagrams. For the twist N case they comprise the local operator vertex and N gluons in the t-channel interacting through all possible s-channel gluons rungs. The integrals in each ladder cell are ordered in LLA such that the transverse momentum in the above cell plays the role of an ultraviolet cut-off for the one below. The Q^2 value, being the greatest momentum in the upper loop attached to the local operator vertex, is the ultraviolet cut-off for the whole diagram. The evolution equation is obtained by taking the derivative of the diagrams with respect to $\log Q^2$. The kernel $\Phi^{a_i a_j, a_i' a_j'}_{\lambda_i \lambda_j, \lambda_i' \lambda_j'}(x_i, x_j; \beta_i, \beta_j)$ is determined by the logarithmic part of the one-loop integral over the transverse parton momentum $k_\perp$.

Generally the longitudinal momenta x_i are not ordered in LLA, but they have to be ordered in DLA to provide a large logarithm for each ladder cell. The x_i variables increase from the smallest values at the local operator vertex to order of unity ones in the lower part of a diagram. In such a kinematics the logarithmic divergencies that occur in every loop when $\beta_i \to 0$ are cut from below by the longitudinal momentum in the upper cell. Thus DLA implies that the loop integrals in the evolution equation (1) are limited by the condition

$$\beta_i, \beta_j \ll x_i, x_j, \tag{2}$$

which means that the momenta below the s-channel rung (x) and above it (β) are of different orders of magnitude. The most singular contribution comes in (1) from the region where both β_i and β_j tend to zero. Momentum conservation allows this only if

$$x_i + x_j \ll \beta_i, \beta_j, \tag{3}$$

that is, $x_i \approx -x_j$ with logarithmic accuracy. Hereafter it is convenient to assume the momenta directed upward to be positive, and those directed downward to be negative.

In the logarithmic domain the kernel of the evolution equation in DLA can easily be obtained by keeping the terms most singular in β in the gluon–gluon kernel,

$$\Phi^{a,b,c,d}_{\lambda_1,\lambda_2,\lambda_1',\lambda_2'}(x_1, x_2; \beta_1, \beta_2) = 2\delta_{\lambda_1\lambda_2}\delta_{\lambda_1',\lambda_2'} i f^{acg} i f^{bgd}\, x_1 \delta(x_1 - x_2)\frac{1}{\beta_1\beta_2}. \tag{4}$$

Here λ_i, λ_i' are the two dimensional transverse helicity indices, the f^{abc} are the structure constants of the $SU(N_c)$ group. The momenta x_1, x_2, are equal in DLA. They are positive but have opposite directions; one of them is incoming from below, the other is outgoing. The low-scale momenta $\beta_{1,2}$ are not supposed to be equal in DLA since the momentum transfer from below $x_1 - x_2$ is small only compared to the large momenta $x_{1,2}$ but is of the same order as the low-scale ones.

All x_i, β_i momenta are supposed to be positive in the DLA kernel (4); the sign is specified with an additional index $\sigma = \{+, -\}$. Thus each gluon in the structure function is characterized by the color index a, helicity λ, momentum value β and momentum direction σ. The interaction occurs only between gluons with opposite σ.

There are two possible color structures for twist-3 operators – the odderon-like one d^{abc} and the gluon-like one f^{abc}. Both of them go through the equation resulting in an $N_c/2$ factor. This simplifies the color structure of the correlation function, $N^{abc}_{\lambda_1,\lambda_2,\lambda_3} = d^{abc} F_{\lambda_1,\lambda_2,\lambda_3}$ or $N^{abc}_{\lambda_1,\lambda_2,\lambda_3} = f^{abc} F_{\lambda_1,\lambda_2,\lambda_3}$, and the action of the kernel can be written as

$$\begin{aligned} &H_{12} F_{\lambda_1,\sigma_1,\lambda_2,\sigma_2,\lambda_3,\sigma_3}(x_1, x_2, x_3) \\ &= \frac{1}{4}\,\overline{\alpha}\,\delta_{\sigma_1,-\sigma_2}\,\delta_{\lambda_1,\lambda_2}\delta_{\lambda_1',\lambda_2'}\, x_1\,\delta(x_1 - x_2) \\ &\times \int_0^{x_1} \frac{\mathrm{d}\beta_1}{\beta_1} \int_0^{x_2} \frac{\mathrm{d}\beta_2}{\beta_2}\, \big[\, F_{\lambda_1',\sigma_1,\lambda_2',\sigma_2,\lambda_3,\sigma_3}(\beta_1, \beta_2, x_3) \\ &\qquad \pm F_{\lambda_2',\sigma_2,\lambda_1',\sigma_1,\lambda_3,\sigma_3}(\beta_2, \beta_1, x_3)\,\big]\,, \\ &\overline{\alpha} \equiv N_c \frac{\alpha_{\mathrm{S}}}{\pi}, \end{aligned} \tag{5}$$

and similarly for H_{23}, H_{13}. Two terms in the RHS (5) represent the sum of the s- and u-channel diagrams (only even momentum operators survive in this sum for the twist-2 case).

3 Finding the Green function by iterating the Bethe–Salpeter equation

Instead of direct solving of the evolution equation we adopt here another approach similar to reggeon calculus and more suitable to find the asymptotics of the structure function. To this end we rewrite the formal solution of the evolution equation with a given initial condition F_0,

$$F\left(Q^2\right) = \mathrm{e}^{H \log Q^2/\mu^2} F_0$$

through a Mellin transform as

$$F\left(Q^2\right) = \int \frac{\mathrm{d}\nu}{2\pi \mathrm{i}} \left(\frac{Q^2}{\mu^2}\right)^{\nu} \frac{1}{\nu}\, \frac{1}{1 - \frac{1}{\nu} H}\, F_0,$$

where the integral runs along the imaginary axis to the right from all singularities. The equation

$$F(\nu) = F_0 + \frac{1}{\nu} H\, F(\nu) \tag{6}$$

can be treated as the Bethe–Salpeter equation in the theory described by the effective action

$$\begin{aligned} S = &\int \mathrm{d}x\, \Phi^*_{\lambda\sigma}(x)\, \Phi_{\lambda\sigma}(x) \\ &+ \frac{1}{4}\frac{\overline{\alpha}}{\nu} \int \mathrm{d}x\, x\, \frac{\mathrm{d}\beta_1}{\beta_1}\frac{\mathrm{d}\beta_2}{\beta_2}\, \Phi^*_{\lambda_1\sigma_1}(x)\, \theta(x - \beta_1)\, \Phi_{\lambda_2\sigma_2}(\beta_1) \end{aligned}$$

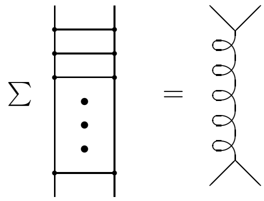

Fig. 1. Two gluon ladder

$$\times\ \delta_{\lambda_1\lambda_2}\delta_{\lambda_3\lambda_4}\left[\delta_{\sigma_1\sigma_2}\delta_{\sigma_3\sigma_4} \pm \delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}\right]\delta_{\sigma_2,-\sigma_4}$$
$$\times\ \Phi^*_{\lambda_3\sigma_3}(x)\,\theta(x-\beta_2)\,\Phi_{\lambda_4\sigma_4}(\beta_2).$$

The solution to (6) is given by the convolution with the Green function,

$$F_{\lambda'_i\sigma'_i}(\nu,\beta_i) \;=\; \int \mathrm{d}x_i\,\varphi_{\lambda_i\sigma_i}(x_i)\,G_{\lambda_i\sigma_i,\;\lambda'_i\sigma'_i}(\nu;x_i,\beta_i) \qquad (7)$$

calculated in the effective theory,

$$G_{\lambda_i\sigma_i,\lambda'_i\sigma'_i}(\nu;x_i,\beta_i) \;=\; \prod_{i=1}^{3}\langle \Phi_{\lambda'_i\sigma'_i}(x_i)\,\Phi^*_{\lambda_i\sigma_i}(\beta_i)\rangle.$$

The initial hadron wavefunction $\varphi_{\lambda_i\sigma_i}(x_i)$ is to be taken at the low Q^2 scale. It cannot be found perturbatively, but its precise form does not influence the large-Q^2 asymptotics. For definiteness we shall consider below the moments; that is, we take

$$\varphi(x_i) = \prod_{i=1}^{3} x_i^{n_i}, \qquad (8)$$

with the integers $n_i \geq\ 0$.

We shall find the Green function by iterating the Bethe–Salpeter equation. We start from the two gluon ladder, or "reggeon", which will be a main building block in the further proceeding. It is schematically shown in Fig. 1, where the solid lines denote the gluon. Iterations of the two-particle kernel results in the matrix

$$\widehat{g}_{\lambda_1\,\lambda_2,\,\lambda'_1\,\lambda'_2}(x_1,x_2;\beta_1,\beta_2) \qquad (9)$$
$$= \frac{1}{2}\,\delta_{\lambda_1\lambda_2}\delta_{\lambda'_1\lambda'_2}\,\frac{1}{\beta_1}\frac{1}{\beta_2}\,\widehat{g}_{12}(\nu;x_1,\{\beta_1,\beta_2\})\,x_1\,\delta(x_1-x_2).$$

Here $\{\beta_1,\beta_2\} \equiv max\{\beta_1,\beta_2\}$, and the matrix g_{ik} acts on the sign variables σ_i, σ_k as follows:

$$\widehat{g}_{ik}(\nu;x,\beta) \equiv g_{\sigma_i\sigma_k,\sigma'_i\sigma'_k}(\nu;x,\beta)$$
$$= \left(\frac{I \pm P_{ik}}{2}A_{ik}\right)_{\sigma_i\sigma_k,\sigma'_i\sigma'_k} g(\nu;x,\beta),$$

where the operator P_{ik} permutes the indices σ_i, σ_k, the matrix A_{ik} permits the interaction only between the partons with opposite momentum signs,

$$(A_{ik})_{\sigma_i\sigma_k,\,\sigma'_i\sigma'_k} \;=\; \delta_{\sigma_i\sigma'_i}\,\delta_{\sigma_k\sigma'_k}\,\delta_{\sigma'_i,-\sigma'_k},$$

and

$$g(\nu;x,\beta) \;=\; \int \frac{\mathrm{d}j}{2\pi \mathrm{i}}\left(\frac{\beta}{x}\right)^{-j}\frac{\overline{\alpha}}{2\nu j-\overline{\alpha}}. \qquad (10)$$

4 Double logarithmic twist-2 anomalous dimension

The expression (9) leads to the usual structure function in the twist-2 case. Indeed, the general form of the twist-2 spin J gluon operator (the F_1 structure function) convoluted with the gauge fixing vector n is

$$n_{\mu_1}\ldots n_{\mu_J}\mathcal{O}_{\mu_1,\ldots,\mu_J} \;=\; (\mathrm{i}\partial)\,A_\nu\;(\mathrm{i}\partial)^{J-1}\,A_\nu.$$

This results in the vertex

$$\delta_{\lambda_1\lambda_2}\beta_1^{m_1}\beta_2^{m_2}\delta(\beta_1+\beta_2),$$

with $m_1 = 1$, $m_2 = J-1$. There is no momentum transfer through the operator, and this is the reason for the momentum delta-function. This vertex should be integrated with the ladder function (where $\overline{\alpha}/2$ is replaced with $\overline{\alpha}$ for the color singlet). The integration has to be done with account of both signs of the $\beta_{1,2}$ variables, which implies the sum over $(+-)$ and $(-+)$ initial sign states. For the $(+-)$ final state, that is, for the positive x_1, we get

$$M_2(J) = -\left[1+(-1)^J\right]\,\delta_{\lambda_1\lambda_2}x_1^J\delta(x_1-x_2)$$
$$\times\int\frac{\mathrm{d}j}{2\pi\mathrm{i}}\,\frac{\overline{\alpha}}{\nu(J-1)-\overline{\alpha}}$$

and the same for the $(-+)$ state. Thus we have reproduced the double logarithmic twist-2 anomalous dimension

$$\gamma_2(J) \;=\; \frac{\overline{\alpha}}{J-1}$$

together with the selection rule allowing only for the even J values.

5 Diagrams for the Green function in (7)

We consider the diagrams for the Green function occurring in (7). The general sum of the three-gluon ladder diagrams can be equivalently presented as a sum of two-gluon ladders ("reggeons") developing between each of the gluon pairs accompanied with a third single gluon as is shown in Figs. 2–4. By employing this representation all diagrams can be summed up in a closed form. The Green function reads

$$G^{\mathrm{tot}}_{d,f} \;=\; \sum_{\{i\},\{i'\}}\overline{G}_{d,f}\left(x_i,\sigma_i,\lambda_i\,|\,\beta_{i'},\sigma_{i'},\lambda_{i'}\right), \qquad (11)$$

where the sum is taken over independent permutations of the incoming and outgoing particles, while the functions $\overline{G}_{d,f}$ stand for the diagrams with a fixed order of the external lines. The symbols d and f label the Green functions for the d^{abc} and f^{abc} color structures. The formula (11) implies simultaneous permutations of all quantum numbers, that is momenta, helicities and colors, which means symmetrization with respect to $\{x_i,\sigma_i,\lambda_i\}$ (or $\{\beta_j,\sigma'_j,\lambda'_j\}$) pair for the d^{abc} tensor and antisymmetrization for the f^{abc} tensor.

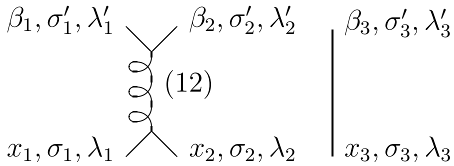

Fig. 2. Diagram for the pair 12 coming from a finite number of iterations

The effective diagrams constructed from the two-gluon ladder and gluon line turns out to be rather simple to calculate the Green function by the direct summation. The result is presented by the sum of three contributions. Two of them are degenerate in the sense that they come from a finite number of iterations. The first one includes the "reggeon" only once. There are three diagrams of this type for three various gluon pairs combined into the ladder. One of them, for the pair 12, is shown in Fig. 2. We have

$$\overline{G}^{(\mathrm{I})}_{d,f}\,(x_i,\beta_i)_{(12)} = \frac{1}{2}\,\delta_{\lambda_1\lambda_2}\delta_{\lambda_3\lambda'_3}\delta_{\lambda'_1\lambda'_2}\,\frac{1}{\beta_1\beta_2}\,\widehat{g}_{12}(\nu;x_1,\{\beta_1,\beta_2\}) \times\, x_1\delta(x_1-x_2)\delta(x_3-\beta_3)\,\delta_{\sigma_3\sigma'_3}.$$

The others can be obtained by permutations of the indices $(123)\to(231)$ and $(123)\to(132)$.

The second contribution arises from the diagram with the two "reggeons". Figure 3 presents the diagram where the gluon pair 12 switches to the pair 23,

$$\overline{G}^{(\mathrm{II})}_{d,f}\,(x_i,\beta_i)_{(23)(12)} = \frac{1}{4}\,\delta_{\lambda_2\lambda_3}\delta_{\lambda_1\lambda'_3}\delta_{\lambda'_1\lambda'_2}\,\frac{1}{\beta_1\beta_2\beta_3}\,\widehat{g}_{23}(\nu;x_1,\{\beta_1,\beta_2\}) \times\,\widehat{g}_{12}(\nu;x_2,\{x_1,\beta_3\})\,x_2\delta(x_2-x_3). \tag{12}$$

The other five terms result in this case from (12) after independent permutations $(123)\to(231)$ and $(123)\to(132)$ of the upper and lower (in the sense of Fig. 3) indices but excluding the equal ones. In other words, the sum is taken over various ways to combine the incoming and outgoing gluons in different two-particle ladders.

The contributions starting with the three "reggeons" develop a regular series which can be written as

$$\overline{G}^{(\mathrm{III})}_{d,f}\,(x_i,\beta_i)_{(12)(12)}$$

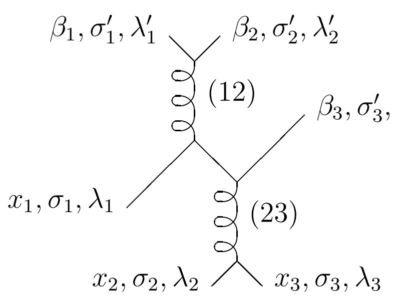

Fig. 3. Diagram where the gluon pair 12 switches to the pair 23

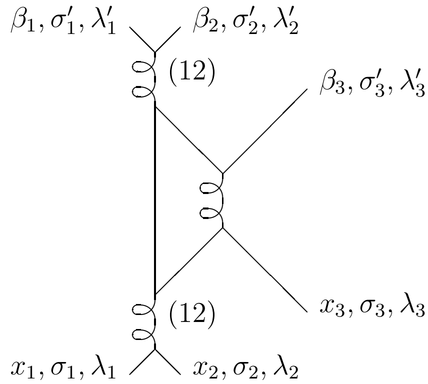

Fig. 4. Diagram corresponding either to the first term in (14) or to all terms in (14) if the middle "reggeon" is replaced with the full function $W_{d,f}$

$$= \frac{1}{4}\,\delta_{\lambda_1\lambda_2}\delta_{\lambda_3\lambda'_3}\delta_{\lambda'_1\lambda'_2}\,\frac{1}{\beta_1\beta_2\beta_3}\,\widehat{g}_{12}(\nu;x_1,x_3) \times \int\frac{\mathrm{d}\beta}{\beta}\,W_{d,f}(x_3,\nu;\{\beta_3,\beta\})\,\widehat{g}_{12}(\nu;\beta,\{\beta_1,\beta_2\}), \tag{13}$$

$$W_{d,f}(\nu;x,\beta) = \frac{1}{2}\,\widehat{g}(\nu;x,\beta)+\frac{1}{4}\int\frac{\mathrm{d}\beta'}{\beta'}\widehat{g}\,(\nu;x,\beta')\,\widehat{g}\,(\nu;\beta',\beta)+\ldots \tag{14}$$

Figure 4 shows the diagram corresponding either to the first term in (14) or to all terms in (14) if the middle "reggeon" is replaced with the full function $W_{d,f}$.

There are eight other terms besides (13), which can be obtained from it by independent permutations $(123)\to(231)$ and $(123)\to(132)$ of the upper and lower indices in Fig. 4.

The Mellin transform (10) turns the convolutions over β' into the usual products, making the series (14) equivalent to the purely matrix problem,

$$W = \frac{1}{2}\frac{\overline{\alpha}}{\nu j}H + \left(\frac{1}{2}\frac{\overline{\alpha}}{\nu j}H\right)^2 + \ldots = H\,\frac{1}{2\frac{\nu j}{\alpha}-H}, \tag{15}$$

where the "reggeons" "dissociate" into a two-particle interaction,

$$H = \sum H_{i,i+1}, \qquad H_{i,k} = \frac{1}{2}\,\delta_{\lambda_i\lambda_k}\delta_{\lambda'_i\lambda'_k}\,\frac{1\pm P_{ik}}{2}\,A_{ik},$$

and the problem is reduced to the inversion of a finite matrix.

The above iterations exhibit that at each step two momenta with opposite directions have the same value much larger than the value of the third momentum ($x_{2,3}\gg x_1$ in Fig. 3 and $x_{1,2}\gg x_3$ in Fig. 4). This property expresses the longitudinal momentum conservation within the DLA accuracy – the sum of all momenta is small compared to their natural scale.

6 Contributions to $G_{d,f}$

The Green function is convoluted over variables β_i and helicities indices λ'_i with the operator vertex given by the expression

$$O_{\lambda'_1,\mu_1,\lambda'_2,\mu_2,\lambda'_3,\mu_3}(\beta_1,\beta_2,\beta_3) \tag{16}$$
$$= \Gamma_{\lambda'_1,\mu_1,\lambda'_2,\mu_2,\lambda'_3,\mu_3}\, \beta_1^{m_1}\beta_2^{m_2}\beta_3^{m_3}\delta(\beta_1+\beta_2+\beta_3),$$

where the longitudinal δ-function corresponds to forward kinematics without momentum transfer. Taking then the moments with respect variables x_i (8) we get the Green function in the moments' representation, $G^{\rm tot}_{d,f}(m_i,n_i)$. Note that the integrals over x_i, β_i imply positive as well as negative values of the momenta. The negative values are described through the sign variables $\sigma_i = \pm$, for example, the configuration where $\beta_1 < 0$, $\beta_{2,3} > 0$ is associated with the state $(-,+,+)$ and similarly for x_i. The integral over all sign configurations is recovered by the sum over all initial σ'_i and final σ_i values. As a result the Green function written in terms of the moments takes into account both signs of β_i and x_i and does not contain the auxiliary variables σ_i,

$$G^{\rm tot}_{d,f}(m_{i'},n_i)$$
$$= \sum_{\{i'\},\{i\}} \delta_{\lambda_{i_1}\lambda_{i_2}}\delta_{\lambda_{i'_1}\lambda_{i'_2}}\delta_{\lambda_{i_3}\lambda_{i'_3}}$$
$$\times\, \overline{G}_{d,f}\left(m_{i'_1},m_{i'_2},m_{i'_3};n_{i_1},n_{i_2},n_{i_3}\right). \tag{17}$$

The sum here means the independent symmetrization with respect the pairs $\{m_{i'},\lambda_{i'}\}$ and $\{n_i,\lambda_i\}$ for d^{abc} and antisymmetrization for the f^{abc} structures. The helicities λ'_i should be convoluted with the tensor $\Gamma_{\lambda'_i,\mu_i}$ specifying the operator vertex (16).

Separating the common factors, the functions $\overline{G}_{d,f}(m_i, n_i)$ take the form

$$\overline{G}_{d,f}(m_i,n_i)$$
$$= \left[1+(-1)^{m+n}\right]\left((-1)^{n_1}\pm(-1)^{n_2}\right) \tag{18}$$
$$\times\left[\frac{(-1)^{m_1}}{m_2}\pm\frac{(-1)^{m_2}}{m_1}\right]\frac{1}{m+n+2}\,G_{d,f}(m_i,n_i),$$
$$m\equiv m_1+m_2+m_3,\qquad n\equiv n_1+n_2+n_3,$$

where $+$ and $-$ stand for d and f structures, respectively, and the function $G_{d,f}$ is expressed in terms of the three contributions considered above,

$$G_{d,f} = G^{(\rm I)}_{d,f} + G^{(\rm II)}_{d,f} + G^{(\rm III)}_{d,f}.$$

The first contribution yields

$$G^{(\rm I)}_{d,f}(m_i,n_i) = \frac{3}{4}\,\frac{\overline{\alpha}}{2\nu j-\overline{\alpha}}, \tag{19}$$
$$j = m+n_3,$$

while the second one takes the form

$$G^{(\rm II)}_{d}(m_i,n_i) = \frac{3}{8}\,\frac{\overline{\alpha}}{2\nu(m-1)-\overline{\alpha}}\,\frac{\overline{\alpha}}{2\nu j-\overline{\alpha}}, \tag{20}$$

$$G^{(\rm II)}_{f} = 0. \tag{21}$$

($G^{(\rm II)}_{f}$ vanishes after antisymmetrization over the end points.) The third contribution with the matrix (15) inverted reads

$$G^{(\rm III)}_{d} = \frac{3}{8}\,\frac{\overline{\alpha}}{2\nu(m-1)-\overline{\alpha}}\,\frac{\overline{\alpha}}{2\nu j-\overline{\alpha}}\,\frac{\overline{\alpha}}{4\nu(m-1)-3\overline{\alpha}}, \tag{22}$$

$$G^{(\rm III)}_{f} = \frac{3}{8}\,\frac{\overline{\alpha}}{2\nu(m-1)-\overline{\alpha}}\,\frac{\overline{\alpha}}{2\nu j-\overline{\alpha}}\,\frac{\overline{\alpha}}{4\nu(m-1)-\overline{\alpha}}. \tag{23}$$

7 Asymptotic behavior of the structure function

The asymptotic behavior of the structure function for Bjorken variable $x_{\rm B}\to 0$ is determined by the rightmost singularity in the variable J, which has the meaning of local spin operator continued to the complex plane. The spin of the quasipartonic operator is $J = m$; therefore, one needs to continue the function $\overline{G}_{d,f}(m_i,n_i)$ to $m\to 1$ formally keeping the other variables, m_i, n_i, fixed. Because of the signature-like factors $(-1)^{m+n}$ the terms with even or odd m are to be treated separately. Note that in a general LLA case this continuation is non-trivial since the mixing matrix describing the evolution has a rank depending on J [8]. The explicit form of the DLA solutions (19)– (23) makes the continuation much more simple and straightforward. The obtained results show that the anomalous dimension for the d^{abc} color structure is

$$\gamma_d(J) = \frac{3}{4}\,\frac{\overline{\alpha}}{J-1}. \tag{24}$$

The main singularity for the f^{abc} structure is actually given by the pole of the two-gluon state,

$$\gamma_f(J) = \frac{1}{2}\,\frac{\overline{\alpha}}{J-1}. \tag{25}$$

The contribution of the "developed" three-gluon ladder is weaker in this channel, $\gamma'_f(J) = \gamma_f(J)/2$. The contributions of the other singularities are strictly speaking beyond the DLA accuracy, since they produce the extra positive powers of $x_{\rm B}$.

The DLA anomalous dimension (24) is smaller compared to those which can be derived from the direct solution of the BFKL equation obtained in [4,6,7,9]. A possible reason for this is that the known BFKL solutions found for the odderon do not really correspond to quasipartonic operators of twist 3. In this case the DLA result (24) could indicate the existence of another solution of quasipartonic type.

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